MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All Sections / All Groups)

7 OCTOBER 2016

3.00 PM – 5.00 PM

(2 Hours)

GENERAL INSTRUCTIONS:

1. This exam paper consists of **5 pages** with **5 questions only**.
2. Each question is worth **20 marks. Attempt ALL questions.**
3. A normal distribution table is provided in the Appendix.
4. Write all your answers in the answer booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1

(a) The following table shows the sales of a company by region during a particular year. **Copy and complete the table, and draw its corresponding pie chart.** Use approximate sizes for each sector (but indicate the correct angles rounded to units).

[7 marks]

Region	Sales (in millions of $)	Relative Frequency	Percentage
Europe	250		
Africa	132		
Asia	351		
America	267		
TOTAL			

(b) The hours of sunlight in the Arctic Circle was measured for 20 days in January, yielding the following data:

1.7	2.1	3.4	3.6	3.9
2.7	4.6	3.2	3.1	5.5
3.9	2.6	2.1	3.0	1.7
4.5	5.9	2.2	5.2	3.5

(i) Using Sturges' Rule, estimate the number of classes required. [2 marks]

(ii) Copy and complete the frequency distribution table given below: [5 marks]

Class	Class Boundaries	Class Midpoint, m	Tally	Frequency, f	Cumulative Frequency	mf
1.5 – 2.4						
2.5 – 3.4						
3.5 – 4.4						
⋮						

(iii) What are the estimated mean and median for the daily hours of sunlight at the Arctic Circle during the month of January?

[6 marks]

Continued...

Question 2

(a) 260 engineers were surveyed on their familiarity with the programming languages Java and C++. The following table summarizes their responses:

Programming Language(s)	Number of Engineers
Java	165
C++	200
Java and C++	x
Neither	25

(i) Draw a Venn diagram representing the above information. [3 marks]

(ii) Determine x, the number of engineers who are familiar with both Java and C++. [2 marks]

(b) Given the digits 0, 2, 3, 5, 7 and 9, how many **4 digit** numbers can be formed (without repeating digits), if

(i) the number must be odd? [3 marks]

(ii) the numbers must be even and less than 4000? [4 marks]

[Hint: A number should *not* start with a leading zero]

(c) Ken travels to work by car, bus or train. He drives about 3 days out of 10 and takes the bus about 9 days out of 20. The rest of the time he takes the train. If Ken goes by car, there is a 45% chance he will be late for work. With the bus and train, the chances he will be late are 23% and 3% respectively.

(i) Draw the probability tree representing this information. [4 marks]

(ii) Given that Ken is late on a particular day, find the probability that he drove to work. [4 marks]

Question 3

(a) The probability mass function, $f(y)$, of a discrete random variable, Y, is tabulated below:

y	0	1	2	3
$f(y)$	h	i	j	k

(i) Find the values of h, i, j and k if it is known that $P(Y \leq 2) = 0.9$, $P(Y < 2) = 0.5$ and $E[Y] = 1.5$. [8 marks]

(ii) Hence, compute the variance of Y. [2 marks]

Continued...

(b) The voltage (in Volts) at the output of a microphone is a continuous random variable X with probability density function

$$f(x)=\begin{cases} lx^2, & 0 \le x \le 2, \\ 0, & \text{otherwise}, \end{cases}$$

where l is a constant.

(i) Show that $l = 3/8$. [3 marks]

(ii) Compute $P(0.5 \le X \le 1.5)$. [3 marks]

(iii) Find the average output voltage. Hence, evaluate $E[2X+3]$. [4 marks]

Question 4

(a) A soft-drink machine is programmed so that it dispenses an average of $\mu = 200$ milliliters (ml) of liquid per cup. The amount of liquid is normally distributed with a standard deviation of $\sigma = 15$ ml.

(i) What is the probability that a cup contains between 191 ml and 209 ml of liquid? (**Note**: A table of the cumulative standard normal distribution is provided in the appendix.) [4 marks]

(ii) Suppose a cup's maximum capacity is 230 ml. How many cups will probably overflow for the next 1000 drinks? [4 marks]

(b) The number of customers arriving at a car repair workshop is assumed to follow a Poisson distribution with an average rate of 3 customers per hour.

(i) Find the probability that more than 2 customers will arrive in the next hour. [4 marks]

(ii) Find the probability that no customer arrives in the next *two* hours. [2 marks]

(c) Suppose that airplane engines operate independently and malfunction with probability equal to 0.2. Assuming that a plane makes a safe flight if at least half the number of its engines run, determine whether a 4-engine plane or a 2-engine plane has the higher probability for a successful flight. [6 marks]

Continued...

Question 5

(a) A certain brand of LED bulb has an average lifespan of 8 years with a standard deviation of 1.4 years. For a random sample of 100 such bulbs,

(i) find the probability that the mean lifespan of the sample is between 6.8 and 7.5 years. [6 marks]

(ii) find the value of c such that the sample's mean lifespan has only a 15% chance of exceeding c. [4 marks]

(b) 3% of the trees in a plantation are known to have a certain disease.

(i) Let $\hat{p}$ be the proportion of trees that are diseased in a random sample of n trees. What is the sampling distribution for $\hat{p}$ when $n = 100$? What about $n = 200$? Justify your answers. [6 marks]

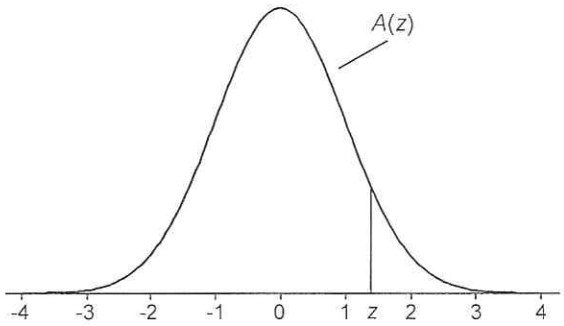
(ii) What is the probability that at least 190 trees in a sample of 200 trees are **free** of the disease? [4 marks]

End of paper.

Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							